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General analysis of direct dark matter detection: From microphysics to observational signatures

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Beginning with a set of simplified models for spin-0, spin-$\frac{1}{2}$, and spin-1 dark matter candidates, we derive the full set of nonrelativistic operators and nuclear matrix elements relevant for direct detection of dark matter and use these to calculate rates and recoil spectra for scattering on various target nuclei. This allows us to explore what high energy physics constraints might be obtainable from direct detection experiments, what degeneracies exist, which operators are ubiquitous, and which are unlikely or subdominant. We find that there are operators which are common to all spins as well operators which are unique to spin-$\frac{1}{2}$ and spin-1 and elucidate two new operators which have not been previously considered. In addition we demonstrate how recoil energy spectra can distinguish fundamental microphysics if multiple target nuclei are used. Our work provides a complete road map for taking generic fundamental dark matter theories and calculating rates in direct detection experiments. This provides a useful guide for experimentalists designing experiments and theorists developing new dark matter models.

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I. INTRODUCTION

The existence of nonbaryonic dark matter has been inferred from measurements including galactic rotation curves [1], large scale structure surveys [2–4], X-ray observations [5], gravitational lensing [6,7], and cosmic microwave background anisotropy measurements [8], spanning cosmological eras from the present day to the remote past. This widespread and robust data has led to cold dark matter models with a cosmological constant, labeled ΛCDM becoming entrenched as the standard cosmological model.

Nevertheless, this impressive array of observations has only been sensitive to the *gravitational* influence of dark matter and constrained its relic abundance, leaving its particle nature as one of the most important open questions in physics. The search for dark matter includes indirect astrophysical searches ([9–13]), collider production efforts (for some examples of dark matter searches at the LHC, see [14–18]) which will examine new territory soon with LHC run 2 which will commence this year, and attempts to observe dark matter interactions with standard model (SM) particles via dark matter-nucleus scattering processes in direct detection experiments, to which we now turn.

The search for dark matter via direct detection goes back at least three decades [19,20] and has been particularly vigorous over the last decade or so with experiments such as LUX [21], Xenon100 [22], CDMS II (Ge) [23], CDMS I (Si) [24], DAMA/LIBRA [25], COGENT [26], and CRESST [27] pushing ever deeper into weakly interacting dark matter mass and scattering cross-section parameter space, but has thus far failed to yield a convincing signal. In the near future detectors such as Super CDMS [28] (which has recently released its first results on low mass dark matter searches [29,30]), XENON1T [31], and DARWIN [32] are expected to push the limits of direct detection orders of magnitude below the current levels.

In order to connect observations to microphysical models one needs a general framework within which to interpret the observations of direct detection experiments. For quite some time the prevailing method of analyzing dark matter-nucleus interactions has been to assume that dark matter is a weakly interacting massive particle (WIMP), and then to categorize the interactions as elastic and isospin conserving and either spin-independent or spin-dependent [33,34]. For some well-studied models of dark matter, such as the weakly interacting Majorana neutralino found in supersymmetry models, this assumption is reasonable.

With an absence of observed dark matter signals, there has of late been a surge in interest in exploring more general types of interactions between dark matter and nuclei. Generalizations include inelastic and momentum dependent interactions, which may arise due to additional structures in the dark sector including excited dark matter states, or dark gauge bosons giving rise to electric and magnetic form factors [35–44].

The formalism of choice for many of these investigations is relativistic effective field theory, which provides a model independent framework to analyze dark matter-SM interactions [45–49]. It has been shown that these effective theories break down when applied to high-momentum transfer experiments, such as the LHC [50,51]. Therefore analyses moved beyond this framework and have moved to what are labeled as "simplified models" instead [52–54]. Simplified models are field theories which extend the SM by a single dark matter particle and a single mediator particle which allows the WIMP to communicate with quarks and/or leptons. The newly added dark matter content is assumed to be a singlet under the SM gauge groups (we will consider some cases where the particles mediating the interaction have SM charge). In this context it is then possible to calculate collider amplitudes valid at the high energies of interest in such experiments. Given this simple dark sector, one can write down an exhaustive list of every combination of WIMP and mediator spins, and all possible tree level interactions. These simplified models have now gained popularity for analyzing indirect detection signals [55,56], allowing connections to be made with the growing body of literature which make use of them.

Another step toward placing dark matter-nucleus interactions on a general footing has been accomplished recently by utilizing a nonrelativistic effective field theory (EFT) approach [57–60]. Since the interactions in direct detection scenarios are assumed to take place due to an incoming dark matter particle with a typical velocity $\mathcal{O}(100\ \mathrm{km/s})$, the recoil momenta in such an interaction will be $\mathcal{O}(\lesssim 100\ \mathrm{keV})$. The particle masses involved, including the nucleons of roughly GeV scale, the dark matter particles, which typically range from the GeV region to several orders of magnitude above, and mediators that can also be quite heavy compared to the typical interaction momenta, produce a situation where an EFT treatment is quite natural.

In order to circumvent as much model dependence as possible, one can construct general interactions which obey Galilean invariance, T-symmetry, and Hermiticity. These operators will take the standard effective four-particle interaction form, reminiscent of Fermi's original model of weak interactions. The nonrelativistic interactions can be shown to be functions of only four parameters including the nucleon spin S_N, the dark matter spin S_χ, the momentum transfer, $\vec{q}$, and a kinematic variable $\vec{v}^\perp$ which is a function of the relative incoming $(\vec{v}_{\chi,\mathrm{in}} - \vec{v}_{N,\mathrm{in}})$ and outgoing velocities $\vec{v}_{\chi,\mathrm{out}} - \vec{v}_{N,\mathrm{out}}$

$$\begin{aligned} \vec{v}^\perp &= \frac{1}{2}(\vec{v}_{\chi,\mathrm{in}} - \vec{v}_{N,\mathrm{in}} + \vec{v}_{\chi,\mathrm{out}} - \vec{v}_{N,\mathrm{out}}) \\ &= \vec{v}_{\chi,\mathrm{in}} - \vec{v}_{N,\mathrm{in}} + \frac{\vec{q}}{2\mu_N} \end{aligned} \tag{1}$$

which obeys $\vec{v}^\perp \cdot \vec{q} = 0$. It was demonstrated in [58] that there exist fifteen such nonrelativistic interactions which arise from twenty possible bilinear combinations of dark matter and nucleons.

The formalism developed in [58] is unique in being the only analysis to comprehensively develop the nuclear physics of direct detection experiments. From this general framework it is now apparent that there are interactions beyond the standard spin independent/dependent type. The origins of these "new" interactions are not necessarily exotic and it has been shown, in the context of relativistic EFT, how many of them can be generated [61].

What has been lacking to date however, is a completely general and comprehensive treatment that connects high energy microphysics with low-energy effective nuclear matrix elements in a model independent way. It is possible, for example, that the various interactions listed in [58] can give rise to degeneracies where different fundamental dark matter Lagrangians, describing dark matter and interaction mediators of various spins, can produce the same interaction types. This will obviously pose problems for attempts to discern the properties of dark matter when interpreting the results of experimental data. Furthermore, dark matter may not be spin-$\frac{1}{2}$, which creates a need for extending the parametric framework from the four descriptors listed above. In particular, as we shall show, this allows the existence of new nonrelativistic operators to appear in the low energy effective theory.

Motivated by the above we present here a general analysis covering a broad spectrum of particle and interaction types, starting from the microphysics, which will enable one to link experiment with fundamental theory while incorporating the new nuclear responses described in [58].

In this work we build upon the nonrelativistic (NR)-EFT description by examining simplified models with generalized Lagrangians for scalar, spinor, and vector dark matter interacting with nucleons via scalar, spinor, and vector mediators, consistent with Lorentz invariance and Hermiticity while imposing stability of the dark matter candidates. We integrate out the heavy mediator and obtain effective relativistic interaction Lagrangians. Next, we take the nonrelativistic limit of these Lagrangians, and identify them with the NR operators from [58], which are reproduced

TABLE I. List of NR effective operators described in [58].

$\mathcal{O}_1$	$1_\chi 1_N$
$\mathcal{O}_2$	$(\vec{v}^\perp)^2$
$\mathcal{O}_3$	$i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$
$\mathcal{O}_4$	$\vec{S}_\chi \cdot \vec{S}_N$
$\mathcal{O}_5$	$i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$
$\mathcal{O}_6$	$(\frac{\vec{q}}{m_N} \cdot \vec{S}_N)(\frac{\vec{q}}{m_N} \cdot \vec{S}_\chi)$
$\mathcal{O}_7$	$\vec{S}_N \cdot \vec{v}^\perp$
$\mathcal{O}_8$	$\vec{S}_\chi \cdot \vec{v}^\perp$
$\mathcal{O}_9$	$i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$
$\mathcal{O}_{10}$	$i\frac{\vec{q}}{m_N} \cdot \vec{S}_N$
$\mathcal{O}_{11}$	$i\frac{\vec{q}}{m_N} \cdot \vec{S}_\chi$
$\mathcal{O}_{12}$	$\vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$
$\mathcal{O}_{13}$	$i(\vec{S}_\chi \cdot \vec{v}^\perp)(\frac{\vec{q}}{m_N} \cdot \vec{S}_N)$
$\mathcal{O}_{14}$	$i(\vec{S}_N \cdot \vec{v}^\perp)(\frac{\vec{q}}{m_N} \cdot \vec{S}_\chi)$
$\mathcal{O}_{15}$	$-(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N})$

below, in Table I. Using these, we identify which electroweak nuclear responses are excited by a given fundamental interaction model and determine the relative importance of various models within the context of direct detection experiments consisting of xenon and germanium targets by exploring the relative magnitude of coefficients of these operators, and also their energy dependence.

The paper is organized as follows; in Sec. II the EFT formalism of [58] is summarized, in Sec. III we build the generalized relativistic Lagrangians and in Sec. IV we outline the signatures and distinguishability of these models in the context of direct detection experiments, providing a framework for both experimentalists and theorists to base their future analyses.

II. EFFECTIVE FIELD THEORY OF DIRECT DETECTION

Conventionally, coherent WIMP-nucleus scattering has been considered to come from two types of interactions; spin-independent (SI) and spin-dependent (SD). SI interactions couple to the charge/mass of the nucleus while SD couples to the spin. The nuclear cross section is generally written in terms of the nucleon cross section at zero momentum transfer, σ_0, and a form factor, $F(q)$, to take into account the loss of coherence over the finite size of the nucleus,

$$\frac{d\sigma}{dE_r} = \frac{M}{2\pi\mu_{\chi M} v^2}(\sigma_0^{\mathrm{SI}} F_{\mathrm{SI}}^2(q) + \sigma_0^{\mathrm{SD}} F_{\mathrm{SD}}^2(q)), \qquad (2)$$

where M is the mass of the target nucleus and $\mu_{\chi M}$ is the WIMP-nucleus reduced mass. This picture has recently been shown to be incomplete, as it is also possible for the WIMP to couple to the nucleus through additional nuclear responses [58,59]. Working in the language of a NR effective field theory Fitzpatrick *et al.* identified 15 operators to characterize the ways in which a WIMP can couple to the various nuclear responses. These operators are constructed from combinations of nonrelativistic vectors which respect Galilean invariance, T symmetry and which are Hermitian. We list them in Table I. The Hermitian vectors are

$$i\frac{\vec{q}}{m_N}, \qquad \vec{v}^{\perp} = \vec{v} + \frac{\vec{q}}{2\mu_N}, \qquad \vec{S}_\chi, \qquad \vec{S}_N, \qquad (3)$$

where $\vec{q} = \vec{p}' - \vec{p} = \vec{k} - \vec{k}'$ is the momentum transfer, $\vec{v}$ is the velocity of WIMP with respect to the nucleus of the detector, μ_N is the reduced mass of the system and $\vec{S}_\chi$ and $\vec{S}_N$ are the WIMP and nuclear spins, respectively. Throughout the paper, we denote by $\vec{p}$ and $\vec{p}'$ the incoming and outgoing WIMP momenta and by $\vec{k}$ and $\vec{k}'$ the incoming and outgoing nuclear momenta, respectively. Energy-momentum conservation implies the orthogonality condition $\vec{q}\cdot\vec{v}^{\perp} = 0$.

As we shall describe, in the following analysis we discovered that two additional NR operators are required to fully describe the scattering of spin-1 WIMPs off nuclei,

$$\mathcal{O}_{17} \equiv i\frac{\vec{q}}{m_N}\cdot\mathcal{S}\cdot\vec{v}_{\perp},$$
$$\mathcal{O}_{18} \equiv i\frac{\vec{q}}{m_N}\cdot\mathcal{S}\cdot\vec{S}_N, \qquad (4)$$

where $\mathcal{S}$ is the symmetric combination of polarization vectors. Together these 17[1] operators form a generalized NR interaction Lagrangian:

$$\mathcal{L}_{\mathrm{NR}} = \sum_{\alpha=n,p}\sum_{i=1}^{15} c_i^\alpha \mathcal{O}_i^\alpha, \qquad (5)$$

where the coefficients c_i^α are given by the microphysics of the interaction and in general one could allow for isospin violation by having different couplings to neutrons and protons inside the nucleus. In Appendix A, we briefly review the procedure employed in [58] to go from the NR operators to the WIMP-nucleus amplitude. This procedure is then applied to the new vector operators in Appendix B.

III. SIMPLIFIED MODELS FOR DIRECT DETECTION

In order to interpret the results of direct detection data in terms of fundamental dark matter models, it is useful to first explore, in as model-independent way as possible, the full range of possible operators that might contribute to any observed signal. We address this question by mapping out the space of possible WIMP-nucleon interactions using simplified models, where "simplified model" means a single WIMP with a single mediator coupling it to the quark sector. While the simplified models considered here are not full-fledged UV complete models, any complete model of WIMP dark matter is expected to make use of these interactions, with relationships between the interactions, and their couplings determined by issues including renormalizability, symmetries, renormalization group operator mixing, etc.

Previous work [61] demonstrated that using only the simplest SI/SD form factors (even with additional momentum dependence taken into account) can lead one to infer wildly incorrect values of the WIMP mass and cross sections if other operators are in fact relevant. Here we go further by starting with simplified models at the Lagrangian level. This is useful for two reasons; it allows us to better explore which NR operators arise from a broad set of Lagrangians, and also make a connection with the growing body of literature which use simplified models for indirect detection and collider searches. Leading order corrections, including calculations beyond the single particle approach which can exhibit large effects in isospin

[1] $\mathcal{O}_{16}$ is omitted since it is a linear combination of other operators.

violating scenarios, have been examined [62,63], but in this initial study, keeping with the simplified model approach, only single particle nucleon interactions at leading order are considered.

While additional structures must exist to allow renormalizability, in particular for possible massive vector bosons in the dark sector, unless this structure involves additional operators which can be included in the broad set of relevant operators we consider, it is not relevant for the ensuing discussion.

When it comes to interpreting signals, knowing comprehensively how different interactions with different nuclei arise from different UV complete Lagrangian terms will allow us to identify degeneracies relevant for distinguishing competing models. Further, it can also help optimize target selection for maximum discrimination of the UV model parameter space.

In building these simplified models we remain agnostic about the WIMP's spin, and consider dark matter spins of $0, \frac{1}{2}$, and 1. We do however only consider renormalizable interactions between quarks and WIMPs. To ensure a stable WIMP, we assume that the WIMP is either charged under some internal gauge group or a discrete symmetry group (for example Z_2). However, we assume that this gauge charge is not shared by quarks. We will couple the WIMP to the quarks via a heavy mediator in two distinct ways: charged and uncharged mediators, each with all possible spins consistent with angular momentum conservation. The mediator mass is chosen to be the heaviest scale in the problem (and certainly much greater than the momentum exchange which characterizes the scattering process) so that we can integrate it out (see Appendix C for details). One should note that in this process the couplings are fixed at the scale given by heavy mediator. In order to give a complete connection for the couplings from the mediator scale to the hadronic scale where direct detection interactions occur, in principle one should use utilize renormalization group equations arising from loop corrections [64–67]. For example in [67], the authors showed that after running down to the hadronic scale, EFT operators could arise which were not present at the high scale. Once again, while this issue is important when considering the relative magnitude of different operators in specific models, we do not focus here so much on the relative magnitude of the coefficients of different operators, but rather on the detector response for individual EFT operators. Future studies which are concerned with more complete model building, beginning with the framework presented in the current work, should consider such effects.

Integrating out the mediator leads to relativistic effective WIMP-nucleon interactions, whose NR limit can then be examined. In the uncharged mediator case we will consider mediators that are neutral under all SM and WIMP gauge charges, while in the charged case, the mediator must have both WIMP and SM gauge charges. Given the above as a guide, our Lagrangian construction is then constrained only by gauge invariance, Lorentz invariance, renormalizability and Hermiticity. In certain cases which follow, the requirement of Hermiticity demands coupling constants be complex. Unless explicitly noted, the coupling constants are dimensionless and can be assumed to be real.

In the following Lagrangian descriptions, universality of mediator couplings to quark flavors is assumed. Including differing, nonuniversal couplings to quarks would have the effect of varying the couplings of dark matter to neutrons and protons. Nonuniversal couplings would introduce further degeneracies when it comes to determining fundamental Lagrangian parameters, which is an interesting complication to consider, but outside the scope of the current study.

A. Uncharged-mediator Lagrangians

1. Scalar dark matter

We begin with a spin-0 scalar WIMP, S, which has some internal charge to ensure stability, and $S^\dagger$ is its Hermitian conjugate. To have renormalizable interactions, the neutral mediator can only be a scalar or a vector. We denote the scalar mediator by ϕ and the vector mediator by G^μ with field strength tensor $\mathcal{G}_{\mu\nu}$.

The most general renormailzable Lagrangian for scalar mediation consistent with the above assumptions is given by

$$\begin{aligned}\mathcal{L}_{S\phi q} = {}& \partial_\mu S^\dagger \partial^\mu S - m_S^2 S^\dagger S - \frac{\lambda_S}{2}(S^\dagger S)^2 \\ &+ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - \frac{m_\phi\mu_1}{3}\phi^3 - \frac{\mu_2}{4}\phi^4 \\ &+ i\bar{q}\not{D}q - m_q\bar{q}q \\ &- g_1 m_S S^\dagger S\phi - \frac{g_2}{2}S^\dagger S\phi^2 - h_1\bar{q}q\phi - ih_2\bar{q}\gamma^5 q\phi, \end{aligned} \tag{6}$$

where we have suppressed all the SM quark interactions. Similarly, the Lagrangian for vector mediation (up to gauge fixing terms) is

$$\begin{aligned}\mathcal{L}_{SGq} = {}& \partial_\mu S^\dagger \partial^\mu S - m_S^2 S^\dagger S - \frac{\lambda_S}{2}(S^\dagger S)^2 \\ &- \frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} + \frac{1}{2}m_G^2 G_\mu G^\mu - \frac{\lambda_G}{4}(G_\mu G^\mu)^2 \\ &+ i\bar{q}\not{D}q - m_q\bar{q}q \\ &- \frac{g_3}{2}S^\dagger S G_\mu G^\mu - ig_4(S^\dagger\partial_\mu S - \partial_\mu S^\dagger S)G^\mu \\ &- h_3(\bar{q}\gamma_\mu q)G^\mu - h_4(\bar{q}\gamma_\mu\gamma^5 q)G^\mu. \end{aligned} \tag{7}$$

2. Spin-$\frac{1}{2}$ dark matter

If the WIMP has spin-$\frac{1}{2}$ (denoted by χ below), then, as in the scalar WIMP case, mediation will only occur via scalar or vector mediators. The most general renormalizable interactions for the scalar (ϕ) and vector mediator (G_μ) cases, respectively are given below,

$$\mathcal{L}_{\chi\phi q} = i\bar{\chi}\not{D}\chi - m_\chi\bar{\chi}\chi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - \frac{m_\phi\mu_1}{3}\phi^3 - \frac{\mu_2}{4}\phi^4 + i\bar{q}\not{D}q - m_q\bar{q}q - \lambda_1\phi\bar{\chi}\chi - i\lambda_2\phi\bar{\chi}\gamma^5\chi - h_1\phi\bar{q}q - ih_2\phi\bar{q}\gamma^5 q, \tag{8}$$

$$\mathcal{L}_{\chi Gq} = i\bar{\chi}\not{D}\chi - m_\chi\bar{\chi}\chi - \frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} + \frac{1}{2}m_G^2 G_\mu G^\mu + i\bar{q}\not{D}q - m_q\bar{q}q - \lambda_3\bar{\chi}\gamma^\mu\chi G_\mu - \lambda_4\bar{\chi}\gamma^\mu\gamma^5\chi G_\mu - h_3\bar{q}\gamma_\mu q G^\mu - h_4\bar{q}\gamma_\mu\gamma^5 q G^\mu. \tag{9}$$

3. Spin-1 dark matter

If the WIMP is a massive spin-1 particle, uncharged mediation to the quark sector can occur via a heavy scalar or a vector particle. The general interaction Lagrangian for the scalar mediation case is

$$\mathcal{L}_{X\phi q} = -\frac{1}{2}\mathcal{X}^\dagger_{\mu\nu}\mathcal{X}^{\mu\nu} + m_X^2 X^\dagger_\mu X^\mu - \frac{\lambda_X}{2}(X^\dagger_\mu X^\mu)^2 + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{m_\phi\mu_1}{3}\phi^3 - \frac{\mu_2}{4}\phi^4 + i\bar{q}\not{D}q - m_q\bar{q}q - b_1 m_X\phi X^\dagger_\mu X^\mu - \frac{b_2}{2}\phi^2 X^\dagger_\mu X^\mu - h_1\phi\bar{q}q - ih_2\phi\bar{q}\gamma^5 q. \tag{10}$$

For the case of vector mediation, there are many possible interactions because the Lorentz indices on the vectors afford a more diverse set of terms. The Lagrangian is given by

$$\mathcal{L}_{XGq} = -\frac{1}{2}\mathcal{X}^\dagger_{\mu\nu}\mathcal{X}^{\mu\nu} + m_X^2 X^\dagger_\mu X^\mu - \frac{\lambda_X}{2}(X^\dagger_\mu X^\mu)^2 - \frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} + \frac{1}{2}m_G^2 G_\mu^2 - \frac{\lambda_G}{4}(G_\mu G^\mu)^2 + i\bar{q}\not{D}q - m_q\bar{q}q - \frac{b_3}{2}G_\mu^2(X^\dagger_\nu X^\nu) - \frac{b_4}{2}(G^\mu G^\nu)(X^\dagger_\mu X_\nu) - [ib_5 X^\dagger_\nu\partial_\mu X^\nu G^\mu + b_6 X^\dagger_\mu\partial^\mu X_\nu G^\nu + b_7\epsilon_{\mu\nu\rho\sigma}(X^{\dagger\mu}\partial^\nu X^\rho)G^\sigma + \mathrm{H.c.}] - h_3 G_\mu\bar{q}\gamma^\mu q - h_4 G_\mu\bar{q}\gamma^\mu\gamma^5 q \tag{11}$$

where, for the Lagrangian to be Hermitian, b_6 and b_7 are complex (this implies a new source of CP violation, which will not be considered further here).

B. Charged-mediator Lagrangians

Here we consider the simplest case of mediators that are charged under both the DM internal symmetry group and SM gauge groups. This is motivated by the absence of spin-$\frac{1}{2}$ mediators (s-channel processes) in the previous section. Such a mediator, if neutral, is forbidden by simultaneous requirements of gauge invariance and renormalizability. Dark matter models with mediators endowed with charges from both the DM and SM side have been considered in the literature before [68,69]. The case of a spin-$\frac{1}{2}$ mediator carrying $SU(3)_c$ is also motivated by studies of heavy quark models. This allows unique interactions as we show below. In particular they necessitate a direct interaction between quarks and WIMPs at the level of the Lagrangian.

1. Scalar dark matter

Scalar WIMPs with a charged scalar or vector mediator do not lead to any Lorentz invariant interactions. This is easy to see since both the scalars (or scalar and vector) and the quark are required in the (gauge invariant) interaction, but there is no way to contract the spinor indices consistently if the mediating particle is a scalar or vector. Therefore, the only possibility is that of a spin-$1/2$ mediator, Q, which acts like a heavy quark. The general renormalizable action is given by

$$\mathcal{L}_{SQq} = \partial_\mu S^\dagger\partial^\mu S - m_S^2 S^\dagger S - \lambda_S(S^\dagger S)^2 + i\bar{Q}\not{D}Q - m_Q\bar{Q}Q + i\bar{q}\not{D}q - m_q\bar{q}q - (y_1 S\bar{Q}q + y_2 S\bar{Q}\gamma^5 q + \mathrm{H.c.}), \tag{12}$$

where y_1 and y_2 are again complex.

2. Spin-$\frac{1}{2}$ dark matter

For a spin-$1/2$ WIMP, both a charged scalar and charged vector mediator exchange can lead to novel interactions. The charged scalar is denoted by Φ and the charged vector by V_μ

$$\mathcal{L}_{\chi\Phi q} = i\bar{\chi}\not{D}\chi - m_\chi\bar{\chi}\chi + (\partial_\mu\Phi^\dagger)(\partial^\mu\Phi) - m_\Phi^2\Phi^\dagger\Phi - \frac{\lambda_\Phi}{2}(\Phi^\dagger\Phi)^2 + i\bar{q}\not{D}q - m_q\bar{q}q - (l_1\Phi^\dagger\bar{\chi}q + l_2\Phi^\dagger\bar{\chi}\gamma^5 q + \mathrm{H.c.}), \tag{13}$$

$$\mathcal{L}_{\chi Vq} = i\bar{\chi}\not{D}\chi - m_\chi\bar{\chi}\chi - \frac{1}{2}\mathcal{V}^\dagger_{\mu\nu}\mathcal{V}^{\mu\nu} + m_V^2 V^\dagger_\mu V^\mu + i\bar{q}\not{D}q - m_q\bar{q}q - (d_1\bar{\chi}\gamma^\mu q V^\dagger_\mu + d_2\bar{\chi}\gamma^\mu\gamma^5 q V^\dagger_\mu + \mathrm{H.c.}), \tag{14}$$

where l_1, l_2, d_1, and d_2 are complex.

3. Vector DM

Here again we only have the case of a spin-$\frac{1}{2}$ mediated interaction between vector DM and quarks (again scalar and vector charged mediators are not possible because they do not lead to Lorentz invariant and renormalizable interactions). The general Lagrangian is given by

$$\mathcal{L}_{XQq} = -\frac{1}{2}\mathcal{X}^{\dagger}_{\mu\nu}\mathcal{X}^{\mu\nu} + m_X^2 X^{\dagger}_{\mu}X^{\mu} - \frac{\lambda_X}{2}(X^{\dagger}_{\mu}X^{\mu})^2 + i\bar{Q}\not{D}Q - m_Q\bar{Q}Q + i\bar{q}\not{D}q - m_q\bar{q}q - (y_3 X_\mu \bar{Q}\gamma^\mu q + y_4 X_\mu \bar{Q}\gamma^\mu\gamma^5 q + \mathrm{H.c.}), \tag{15}$$

where y_3 and y_4 are complex.

IV. NONRELATIVISTIC REDUCTION OF SIMPLIFIED MODELS

After integrating out the heavy mediator we replace quark operators with nucleon operators (see Appendix D), take the nonrelativistic limit (see Appendix C), and match onto the operators given in Table I. The results of this calculation are presented in terms of the c_i coefficients from [59], described in Sec. II, facilitating a straightforward computation of amplitudes and rates. The c_i's are given for each of the WIMP spins in Tables II–IV. With this general framework in place we can now easily find the leading order NR operators for each distinct WIMP-nucleus interaction. One can imagine a set of minimal scenarios where only one or two of the interaction terms from our Lagrangians are present and the rest absent. These scenarios will map out a basis set of interactions which UV models are built from. We only consider scenarios that give rise to a nonzero direct detection signal. Each of these scenarios is listed with its leading operators in Table V and with all operators generated in Table VI. Note that in the case of a complex coupling constant we consider purely real and purely imaginary values as separate cases since they produce a distinct set of operators.

As described earlier, we find that it is important to consider operators beyond those incorporated into the standard spin-independent and spin-dependent formalism, i.e., simple models exist in which one would infer an incorrect rate in current experiments by not including these effects. Also importantly, not all of the NR operators are actually generated at leading order; for example, the operators $\mathcal{O}_2$, $\mathcal{O}_3$, $\mathcal{O}_{13}$, and $\mathcal{O}_{15}$ are missing at leading order. Note that we only consider renormalizable Lagrangians, higher order nonrenormalizable operators are presumably further suppressed. We have also not considered the case of kinetic mixing, which could be used to generate anapole interactions [61], because the effective interaction does not arise from one mediator exchange.

While spin independent interactions are a generic feature of direct couplings to quarks in our charged mediator cases,

TABLE II. Nonzero c_i coefficients for a spin –0 WIMP.

	Uncharged Mediator	Charged Mediator
c_1	$\frac{h_1^N g_1}{m_\phi^2}$	$\frac{y_1^\dagger y_1 - y_2^\dagger y_2}{m_Q m_S} f_T^N$
c_{10}	$\frac{-ih_2^N g_1}{m_\phi^2} + \frac{2ig_4 h_4^N}{m_G^2}\frac{m_N}{m_S}$	$i\frac{y_2^\dagger y_1 - y_1^\dagger y_2}{m_Q m_S}\tilde{\Delta}^N$

TABLE III. c_i coefficients for a spin-$\frac{1}{2}$ WIMP.

	Uncharged Mediator	Charged Mediator
c_1	$\frac{h_1^N\lambda_1}{m_\phi^2} - \frac{h_3^N\lambda_3}{m_G^2}$	$\left(\frac{l_2^\dagger l_2 - l_1^\dagger l_1}{4m_\Phi^2} + \frac{d_2^\dagger d_2 - d_1^\dagger d_1}{4m_V^2}\right) f_T^N + \left(-\frac{l_2^\dagger l_2 + l_1^\dagger l_1}{4m_\Phi^2} + \frac{d_2^\dagger d_2 + d_1^\dagger d_1}{8m_V^2}\right)\mathcal{N}^N$
c_4	$\frac{4h_4^N\lambda_4}{m_G^2}$	$\frac{l_2^\dagger l_2 - l_1^\dagger l_1}{m_\Phi^2}\delta^N - \left(\frac{l_1^\dagger l_1 + l_2^\dagger l_2}{m_\Phi^2} + \frac{d_2^\dagger d_2 - d_1^\dagger d_1}{2m_V^2}\right)\Delta^N$
c_6	$\frac{h_2^N\lambda_2 m_N}{m_\phi^2 m_\chi}$	$\left(\frac{l_1^\dagger l_1 - l_2^\dagger l_2}{4m_\Phi^2} + \frac{d_2^\dagger d_2 - d_1^\dagger d_1}{4m_V^2}\right)\frac{m_N}{m_\chi}\tilde{\Delta}^N$
c_7	$\frac{2h_4^N\lambda_3}{m_G^2}$	$\left(\frac{l_1^\dagger l_2 - l_2^\dagger l_1}{2m_\Phi^2} + \frac{d_1^\dagger d_2 + d_2^\dagger d_1}{4m_V^2}\right)\Delta^N$
c_8	$-\frac{2h_3^N\lambda_4}{m_G^2}$	$\left(\frac{l_1^\dagger l_2 - l_2^\dagger l_1}{2m_\Phi^2} - \frac{d_1^\dagger d_2 + d_2^\dagger d_1}{4m_V^2}\right)\mathcal{N}^N$
c_9	$-\frac{2h_4^N\lambda_3 m_N}{m_\chi m_G^2} - \frac{2h_3^N\lambda_4}{m_G^2}$	$\left(\frac{l_1^\dagger l_2 - l_2^\dagger l_1}{2m_\Phi^2} - \frac{d_1^\dagger d_2 + d_2^\dagger d_1}{4m_V^2}\right)\mathcal{N}^N - \left(\frac{l_1^\dagger l_2 - l_2^\dagger l_1}{2m_\Phi^2} - \frac{d_1^\dagger d_2 + d_2^\dagger d_1}{4m_V^2}\right)\frac{m_N}{m_\chi}\Delta^N$
c_{10}	$\frac{h_2^N\lambda_1}{m_\phi^2}$	$i\left(\frac{l_1^\dagger l_2 - l_2^\dagger l_1}{4m_\Phi^2} + \frac{d_2^\dagger d_1 - d_1^\dagger d_2}{4m_V^2}\right)\tilde{\Delta}^N - i\frac{l_1^\dagger l_2 - l_2^\dagger l_1}{m_\Phi^2}\delta^N$
c_{11}	$-\frac{h_1^N\lambda_2 m_N}{m_\phi^2 m_\chi}$	$i\left(\frac{l_2^\dagger l_1 - l_1^\dagger l_2}{4m_\Phi^2} + \frac{d_2^\dagger d_1 - d_1^\dagger d_2}{4m_V^2}\right)\frac{m_N}{m_\chi} f_T^N + i\frac{l_1^\dagger l_2 - l_2^\dagger l_1}{m_\Phi^2}\frac{m_N}{m_\chi}\delta^N$
c_{12}	0	$\frac{l_2^\dagger l_1 - l_1^\dagger l_2}{m_\Phi^2}\delta^N$

it is sometimes possible to suppress them. In the scalar (and vector) WIMP with charged mediator cases, it is possible to suppress the spin independent interaction by ensuring that $|y_1| = |y_2|$ ($|y_3| = |y_4|$) while keeping their relative phases nonzero (or π). While these nonminimal scenarios require some fine tuning we include it for completeness and label them y_1, y_2 and y_3, y_4.

TABLE IV. c_i coefficients for a spin-1 WIMP.

	Uncharged Mediator	Charged Mediator
c_1	$\frac{b_1 h_1^N}{m_\phi^2}$	$\frac{y_3^\dagger y_3 - y_4^\dagger y_4}{m_Q m_X} f_T^N$
c_4	$\frac{4\mathrm{Im}(b_7)h_4^N}{m_G^2} + i\frac{q^2}{m_X^2}\frac{\mathrm{Re}(b_7)h_4^N}{m_G^2} - \frac{q^2}{m_X m_N}\frac{\mathrm{Re}(b_6)h_3^N}{m_G^2}$	$2\frac{y_3^\dagger y_3 - y_4^\dagger y_4}{m_Q m_X}\delta^N$
c_5	$\frac{\mathrm{Re}(b_6)h_3^N}{m_G^2}\frac{m_N}{m_X}$	0
c_6	$\frac{\mathrm{Re}(b_6)h_3^N}{m_G^2}\frac{m_N}{m_X} - i\frac{\mathrm{Re}(b_7)h_4^N}{m_G^2}\frac{m_N^2}{m_X^2}$	0
c_8	$\frac{2\mathrm{Im}(b_7)h_3^N}{m_G^2}$	0
c_9	$-\frac{2\mathrm{Re}(b_6)h_4^N}{m_G^2}\frac{m_N}{m_X} + \frac{2\mathrm{Im}(b_7)h_3^N}{m_G^2}$	0
c_{10}	$\frac{b_1 h_2^N}{m_\phi^2} - \frac{3b_5 h_4^N}{m_G^2}\frac{m_N}{m_X}$	$i\frac{y_4^\dagger y_3 - y_3^\dagger y_4}{m_Q m_X}\tilde{\Delta}^N$
c_{11}	$\frac{\mathrm{Re}(b_7)h_3^N}{m_G^2}\frac{m_N}{m_X}$	$i\frac{y_4^\dagger y_3 - y_3^\dagger y_4}{m_Q m_X}\delta^N$
c_{12}	0	$2i\frac{y_3^\dagger y_4 - y_4^\dagger y_3}{m_Q m_X}\delta^N$
c_{14}	$-\frac{2\mathrm{Re}(b_7)h_4^N}{m_G^2}\frac{m_N}{m_X}$	0
c_{17}	$-\frac{4\mathrm{Im}(b_6)h_3^N}{m_G^2}\frac{m_N}{m_X}$	0
c_{18}	$\frac{4\mathrm{Im}(b_6)h_4^N}{m_G^2}\frac{m_N}{m_X}$	$-2i\frac{y_4^\dagger y_3 - y_3^\dagger y_4}{m_Q m_X}\delta^N$

TABLE V. Leading order operators which can arise from the relativistic Lagrangians considered in this work, the column "$\mathcal{L}$ terms" gives the nonzero couplings for that scenario. Each row represents a possible leading order direct detection signal. A "†" indicates that the mediator is charged. The "Eqv. M_m" column gives the mediator mass required for each scenario to produce ~ 10 events $\mathrm{t}^{-1}\,\mathrm{yr}^{-1}\,\mathrm{keV}^{-1}$ in xenon, with couplings set to 0.1.

WIMP spin	Mediator spin	$\mathcal{L}$ terms	leading NR operator	Eqv. M_m
0	0	h_1, g_1	$\mathcal{O}_1$	14 TeV
0	0	h_2, g_1	$\mathcal{O}_{10}$	16 GeV
0	1	h_4, g_4	$\mathcal{O}_{10}$	9 GeV
0	$\frac{1}{2}^\dagger$	y_1	$\mathcal{O}_1$	3.7 PeV
0	$\frac{1}{2}^\dagger$	y_2	$\mathcal{O}_1$	3.7 PeV
0	$\frac{1}{2}^\dagger$	y_1, y_2	$\mathcal{O}_{10}$	56 GeV
$\frac{1}{2}$	0	h_1, λ_1	$\mathcal{O}_1$	14 TeV
$\frac{1}{2}$	0	h_2, λ_1	$\mathcal{O}_{10}$	330 GeV
$\frac{1}{2}$	0	h_1, λ_2	$\mathcal{O}_{11}$	16 GeV
$\frac{1}{2}$	0	h_2, λ_2	$\mathcal{O}_6$	2.1 GeV
$\frac{1}{2}$	1	h_3, λ_3	$\mathcal{O}_1$	6.8 TeV
$\frac{1}{2}$	1	h_4, λ_3	$\mathcal{O}_9$	6.9 GeV
$\frac{1}{2}$	1	h_3, λ_4	$\mathcal{O}_8$	220 GeV
$\frac{1}{2}$	1	h_4, λ_4	$\mathcal{O}_4$	150 GeV
$\frac{1}{2}$	$0^\dagger$	l_1	$\mathcal{O}_1$	7.6 TeV
$\frac{1}{2}$	$0^\dagger$	l_2	$\mathcal{O}_1$	5.9 TeV
$\frac{1}{2}$	$1^\dagger$	d_1	$\mathcal{O}_1$	6.4 TeV
$\frac{1}{2}$	$1^\dagger$	d_2	$\mathcal{O}_1$	7.2 TeV
1	0	h_1, b_1	$\mathcal{O}_1$	14 TeV
1	0	h_2, b_1	$\mathcal{O}_{10}$	12 GeV
1	1	h_4, b_5	$\mathcal{O}_{10}$	6.0 GeV
1	1	$h_3, b_6^{\mathrm{Re}}(b_6^{\mathrm{Im}})$	$\mathcal{O}_5(\mathcal{O}_{17})$	6.8 GeV (26 GeV)
1	1	$h_4, b_6^{\mathrm{Re}}(b_6^{\mathrm{Im}})$	$\mathcal{O}_9(\mathcal{O}_{18})$	3.1 GeV (5.4 GeV)
1	1	$h_3, b_7^{\mathrm{Re}}(b_7^{\mathrm{Im}})$	$\mathcal{O}_{11}(\mathcal{O}_8)$	210 GeV (280 GeV)
1	1	$h_4, b_7^{\mathrm{Re}}(b_7^{\mathrm{Im}})$	$\mathcal{O}_4(\mathcal{O}_4)$	90 MeV (190 GeV)
1	$\frac{1}{2}^\dagger$	y_3	$\mathcal{O}_1$	3.7 PeV
1	$\frac{1}{2}^\dagger$	y_4	$\mathcal{O}_1$	3.7 PeV
1	$\frac{1}{2}^\dagger$	y_3, y_4	$\mathcal{O}_{11}$	150 TeV

Aside from scalar WIMPs, each particular spin produces some nonrelativistic operators that are unique to that spin. Also, importantly, the operators $\mathcal{O}_1$ and $\mathcal{O}_{10}$ are generic to all spins. In five cases relativistic operators generate unique nonrelativistic operators at leading order. Therefore distinguishing WIMP scenarios in these cases reduces to experimentally discerning between these operators (see also [70]). Given the likely low statistics of any detection in upcoming direct detection experiments, subleading operators are not likely to contribute enough to provide any further discriminating power.

V. OBSERVABLES

The principle observable in direct detection experiments is the differential event rate. Since the incoming WIMPs originate in the galactic halo, one must average over the WIMP velocity distribution, $f(v)$, which we assume for the purposes of this paper to be Maxwell-Boltzmann,

$$\frac{dR}{dE_R} = N_T \frac{\rho_\chi M}{2\pi m_\chi} \int_{v_{\min}} \frac{f(v)}{v} P_{\mathrm{tot}} dv \tag{16}$$

where we use the value $\rho_\chi = 0.3\ \mathrm{GeV/cm^3}$ for the local dark matter density, N_T is the number of nuclei in the target, and P_{tot} can be calculated from the amplitude $\mathcal{M}$ in Eq. (A10)

$$P_{\mathrm{tot}} = \frac{1}{2j_\chi + 1} \frac{1}{2j_N + 1} \sum_{\mathrm{spins}} |\mathcal{M}|^2. \tag{17}$$

Throughout this work we use the *Mathematica* package supplied in [59] to calculate rates. To determine the leading order operator which arises from a given relativistic scenario we first plot the rate for each of the NR operators in xenon-131. To simply compare the operators we set the c_i coefficients to be the same and normalized the overall rate to that of $\mathcal{O}_1$, see Fig. 1. Since operators are either zero, first or second order in momentum transfer q or velocity $\vec{v}^\perp$, the relative strengths of the operators span 16 orders of magnitude. This is an important point to keep in mind when finding the leading operator, as sometimes a term which appears to be higher order in q can dominate the nonrelativistic reduction. For example in the $b_7^{\mathrm{Re}} h_4$ scenario, one finds that $q^2\mathcal{O}_4$ dominates over the $\mathcal{O}_6$ and $\mathcal{O}_{14}$ which contain powers of q within the operators.

Since the Lagrangians we have considered are not tied to specific complete and consistent particle physics models, the mediator masses are not fixed in advance and thus specific event rates are not predicted in advance. Clearly one requires a rate that is low enough to evade the current experimental constraints. For example, a 50 GeV WIMP producing 10 events per tonne per year is sufficiently low to evade the bounds from LUX [21]. For demonstration purposes we set the couplings to 0.1 (or $0.1i$ for imaginary) in the various Lagrangians and find a mediator mass that will produce 10 events/t/y in the signal region for xenon (5–45 keV). The calculated masses are given in Table V. It is perhaps telling that the mediator masses span 6 orders of magnitude, from just a few GeV up to a PeV. While it is unlikely that a full model of thermal relic dark matter could be built around all of these Lagrangians, it is nevertheless a useful metric to estimate the relative strength of the different nuclear responses to each of the operators.

In Figs. 2–5 we have plotted rates for two common targets. For simplicity and again for demonstration purposes, we only plot the rates for a single isotope of both germanium and xenon. The choice of isotopes, ^{73}Ge and ^{131}Xe, was made to ensure sensitivity to spin-dependent responses. As can be

TABLE VI. List of scenarios with leading operators which are distinguishable via the ratio $\frac{dR_{Xe}}{dE} / \frac{dR_{Ge}}{dE}$.

		$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$	$\mathcal{O}_4$	$q^2\mathcal{O}_4$	$\mathcal{O}_5$	$\mathcal{O}_6$	$\mathcal{O}_7$	$\mathcal{O}_8$	$\mathcal{O}_9$	$\mathcal{O}_{10}$	$\mathcal{O}_{11}$	$\mathcal{O}_{12}$	$\mathcal{O}_{13}$	$\mathcal{O}_{14}$	$\mathcal{O}_{15}$	$\mathcal{O}_{17}$	$\mathcal{O}_{18}$
Spin-0 WIMP	(h_1, g_1)	✓																	
	(h_2, g_1)											✓							
	(h_4, g_4)											✓							
	(y_1)	✓										✓							
	(y_2)	✓										✓							
	(y_1, y_2)											✓							
Spin-$\frac{1}{2}$ WIMP	(h_1, λ_1)	✓																	
	(h_2, λ_1)											✓							
	(h_1, λ_2)												✓						
	(h_2, λ_2)							✓											
	(h_3, λ_3)	✓																	
	(h_4, λ_3)								✓		✓								
	(h_3, λ_4)									✓	✓								
	(h_4, λ_4)				✓														
	(l_1)	✓			✓	✓													
	(l_2)	✓			✓	✓													
	(d_1)	✓			✓	✓													
	(d_2)	✓			✓	✓													
Spin-1 WIMP	(h_1, b_1)	✓																	
	(h_2, b_1)											✓							
	(h_4, b_5)											✓							
	(h_3, b_6)					✓	✓	✓										✓[a]	
	(h_4, b_6)										✓								✓[a]
	(h_3, b_7)									✓[a]	✓[a]		✓						
	(h_4, b_7)				✓[a]	✓		✓								✓			
	(y_3)	✓			✓							✓	✓	✓					✓
	(y_4)	✓			✓							✓	✓	✓					✓
	(y_3, y_4)											✓	✓	✓					✓

[a]Purely imaginary scenario for that coupling.

seen in the figures, many operators produce rates with similar recoil energy dependence in the same target, but different nuclei can have very different responses to the various operators [58]. Thus a complementary choice of nuclear targets can provide important discriminating information.

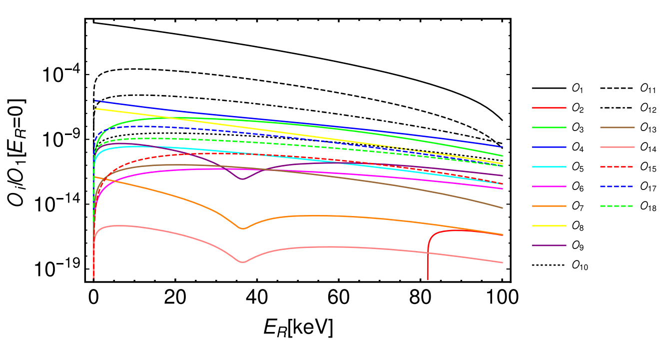

FIG. 1 (color online). The relative strength of event rates for a 50 GeV spin-$\frac{1}{2}$ WIMP in xenon for each of the nonrelativistic operators in Table I, where the coefficients of each operator are set to be equal.

To illustrate this discriminating power we plot the ratio of the rates in xenon and germanium in Figs. 5 and 6. We choose to only present ratios for the uncharged mediator cases of spinor and vector WIMPs since the other cases produce trivial results (all operators being spin independent). To estimate the effect astrophysical uncertainties will have on discriminating between operators, we plot the rate for a range of astrophysical parameters from $v_0 = 200$ m/s, and $v_{\rm esc} = 500$ m/s (lower) to $v_0 = 240$ m/s and $v_{\rm esc} = 600$ m/s (upper). The uncertainty in the dark matter density does not appear since we are considering the ratio of rates. Given the vastly different energy dependence of the ratio of rates of each scenario the astrophysical errors do not completely inhibit their identification. Furthermore, operators $\mathcal{O}_9$ and $\mathcal{O}_{14}$, produced in scenarios $h_4 b_7^{\rm Re}$ and $h_4 b_6^{\rm Re}$, respectively, remain indistinguishable when considering the ratio of rates. While it appears that in principle almost every operator is discernible, in practice isotopically impure targets and low statistics will further complicate the situation and provide limits on practical discrimination.

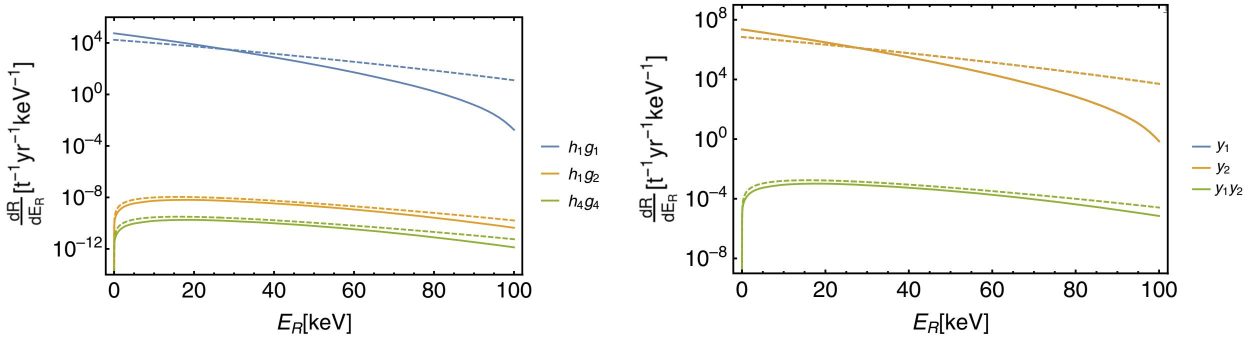

FIG. 2 (color online). Rates for a 50 GeV spin-0 WIMP in xenon (solid) and germanium (dashed) with uncharged (left) and charged mediators (right), assuming mediator mass of 1 TeV and $\mathcal{O}(1)$ coupling constants.

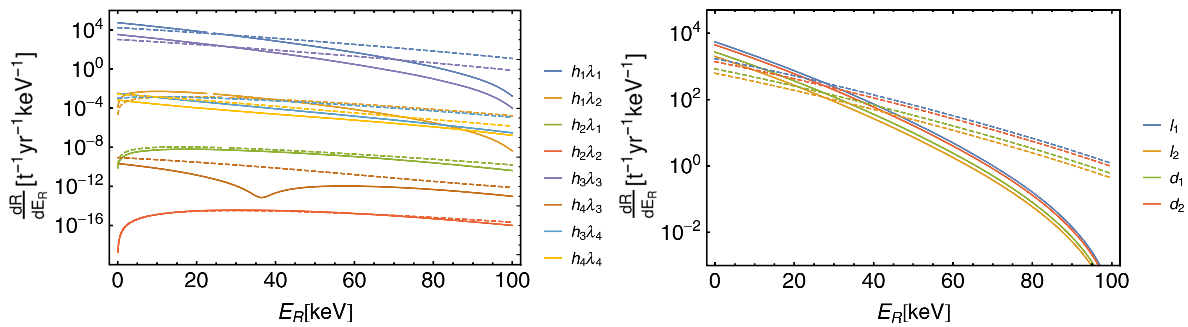

FIG. 3 (color online). Rates for a 50 GeV spin-$\frac{1}{2}$ WIMP in xenon (solid) and germanium (dashed) with uncharged (left) and charged mediators (right), assuming mediator mass of 1 TeV and $\mathcal{O}(1)$ coupling constants.

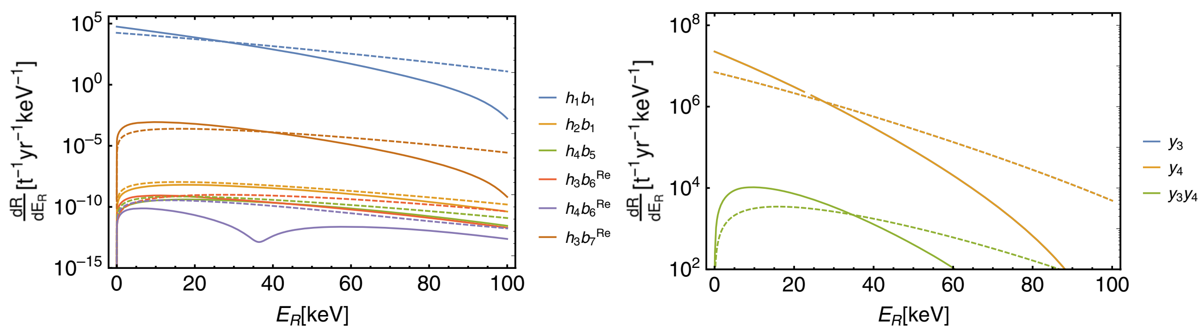

FIG. 4 (color online). Rates for a 50 GeV spin-1 WIMP in xenon (solid) and germanium (dashed) with uncharged (left) and charged mediators (right), assuming mediator mass of 1 TeV and $\mathcal{O}(1)$ coupling constants.

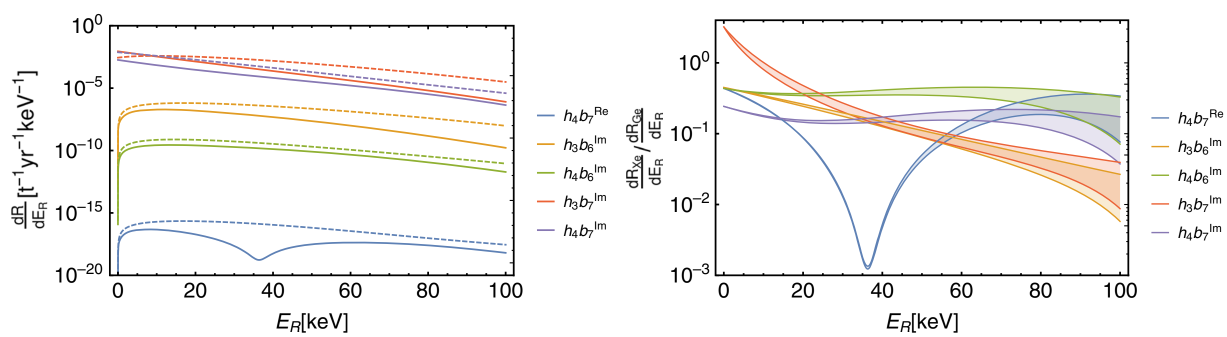

FIG. 5 (color online). Rates (left) for a 50 GeV spin-1 WIMP in xenon (solid) and germanium (dashed) with uncharged mediators and imaginary couplings, assuming mediator mass of 1 TeV and $\mathcal{O}(1)$ coupling constants. Also shown is the ratio of rates in xenon and germanium (right).

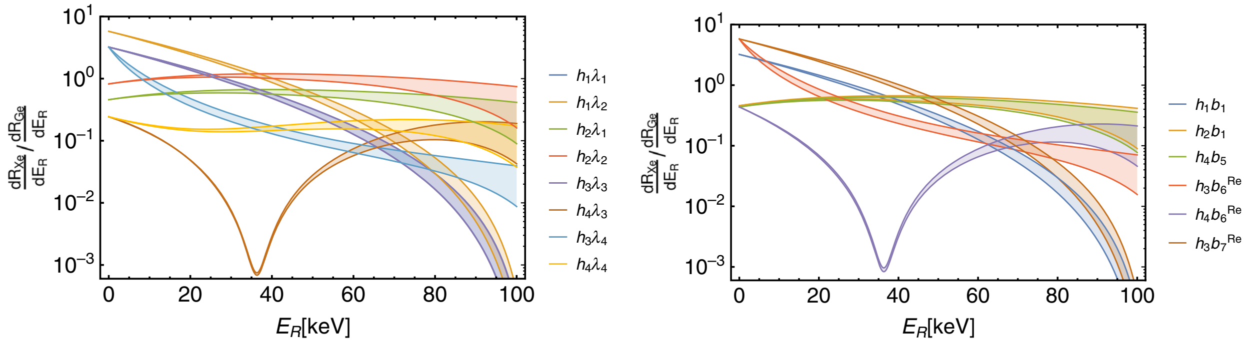

FIG. 6 (color online). Ratio of rates in xenon and germanium, illustrating the discriminating power of having multiple nuclear targets. For a 50 GeV spin-$\frac{1}{2}$ WIMP with uncharged mediator (left) and a 50 GeV spin-1 WIMP with uncharged mediator (right), the shaded regions show the upper and lower bounds due to the astrophysical parameters.

VI. CONCLUSION

The analysis we have given here builds on previous analyses to provide, in generality, a road map to use event rates in direct dark matter detectors to constrain fundamental dark matter models. We have outlined the steps needed to go from fundamental Lagrangians, first to relativistic operators, then to nonrelativistic operators, and finally to produce nuclear matrix elements. In the process several significant facts have been elaborated.

(i) Not all possible nonrelativistic operators contributing to nuclear matrix elements in direct detection will arise from simplified UV complete dark matter models. This is mainly because of Lorentz symmetry, which restricts interactions depending on spin.

(ii) Spinor and vector WIMPs each have NR operators which are unique to their simplified models at leading order, $\mathcal{O}_6$ for spin-$\frac{1}{2}$ and $\mathcal{O}_5$, $\mathcal{O}_{17}$, and $\mathcal{O}_{18}$ for spin-1. The last two, being formed from a symmetric combination of polarization vectors, can only arise for spin-1.

(iii) Two nonrelativistic operators, $\mathcal{O}_1$ and $\mathcal{O}_{10}$, are ubiquitous and arise for all WIMP spins we have explored. They follow at the leading order from the simplest quark bilinears, $\bar{q}q$ and $\bar{q}\gamma^5 q$ or $\bar{q}\gamma^\mu\gamma^5 q$, respectively, which are present in our simplified models for each WIMP spin.

(iv) In 5 of our simplified model scenarios, the leading nonrelativistic operator is not present in any other scenario at leading order.

(v) Two new nonrelativistic operators [Eq. (4)] not previously considered within the context of the full array of allowed nuclear responses arise at low energies if spin-1 WIMP dark matter is allowed for. They arise from the symmetric combination of spin-1 polarization vectors, which is linearly independent of the antisymmetric combination that constitutes the spin vector.

(vi) While not all operators can be distinguished on the basis of their impact on the differential event rates, they can produce radically different energy dependence for scattering off different nuclear targets. We have shown that a complementary use of different target materials (xenon and germanium in this case) allows one to distinguish between different particle physics models of WIMP dark matter.

While current detectors have only yielded upper limits, with new generations of larger detectors with greater energy resolution and lower thresholds coming online, the search for WIMP dark matter has never been so vibrant and promising. The tools we have provided here should help experimenters to probe the most useful parameter space, to interpret any nonzero signals in terms of constraints on fundamental models, and should allow theorists who build fundamental models to frame predictions in an accurate and simple way so that they might be directly compared with experiment.

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APPENDIX A: NONRELATIVISTIC EFFECTIVE FIELD THEORY REVIEW

We briefly outline the details of the nonrelativistic effective field theory of dark matter direct detection, discussed in [58]. They begin with writing down the full nonrelativistic interaction Lagrangian, which in 2-component isospin space is

$$\mathcal{L}_{\rm NR} = \sum_{\tau=0,1} \sum_{i=1}^{15} c_i^\tau \mathcal{O}_i t^\tau \tag{A1}$$

where t^0 and t^1 are the identity matrix and the Pauli matrix σ^3, respectively. The nucleus is composed of nucleons, and these can individually interact with the WIMP. This is incorporated by considering the operator $\mathcal{O}(j)$ as an interaction between a single nucleon, j, and the WIMP, and then summing over the nucleons.

$$\sum_{\tau=0,1}\sum_{i=1}^{15} c_i^\tau \mathcal{O}_i t^\tau \to \sum_{\tau=0,1}\sum_{i=1}^{15} c_i^\tau \sum_{j=1}^{A} \mathcal{O}_i(j) t^\tau(j) \tag{A2}$$

where A is the atomic mass number given by the total number of neutrons and protons. One can do the same reduction with $\vec{v}^\perp$,

$$\begin{aligned}\vec{v}^\perp &\to \{\vec{v}_\chi - \vec{v}_N(i), i = 1, \ldots, A\}\\ &\equiv \vec{v}_T^\perp - \{\dot{\vec{v}}_N(i), i = 1, \ldots, A-1\}\end{aligned} \tag{A3}$$

where $\vec{v}_\chi$ and $\vec{v}_N(i)$ are the symmetrized combination of incoming and outgoing velocities for the WIMP and nucleons, respectively. $\vec{v}_T^\perp$ (here T stands for target, i.e., the nuclear center-of-mass) is defined as

$$\vec{v}_T^\perp = \vec{v}_\chi - \frac{1}{2A}\sum_{i=1}^{A}[\vec{v}_{N,\mathrm{in}}(i) + \vec{v}_{N,\mathrm{out}}(i)]. \tag{A4}$$

This allows for a decomposition of the nucleon velocities into internal velocities $\dot{\vec{v}}_N(i)$ that act only on intrinsic nuclear coordinates and "in" and "out" velocities that evolve as a WIMP scatters off the detector. As an example, the dot product between $\vec{v}_N^\perp$ and $\vec{S}_N$ can be rewritten as

$$\vec{v}^\perp \cdot \vec{S}_N \to \sum_{i=1}^{A}\frac{1}{2}[\vec{v}_{\chi,\mathrm{in}} + \vec{v}_{\chi,\mathrm{out}} - \vec{v}_{N,\mathrm{in}}(i) - \vec{v}_{N,\mathrm{out}}(i)]\cdot \vec{S}_N(i) \tag{A5}$$

$$= \vec{v}_T^\perp \cdot \sum_{i=1}^{A}\vec{S}_N(i) - \left\{\sum_{i=1}^{A}\frac{1}{2}[\vec{v}_{N,\mathrm{in}}(i) + \vec{v}_{N,\mathrm{out}}(i)]\cdot \vec{S}_N(i)\right\}_{\mathrm{int}}. \tag{A6}$$

The second term in the curly brackets is internal to the nucleus and acts as an operator on the "in" and "out" nucleon states. $\vec{v}_{N,\mathrm{in}}$ can be replaced by $\vec{p}_{N,\mathrm{in}}/M$ acting on the incoming state, which can in turn be replaced by $i\overleftarrow{\nabla}/M$, and similarly $\vec{p}_{N,\mathrm{out}}/M$ by $-i\vec{\nabla}/M$ on the outgoing nuclear state. Finally, since the nucleus is nonzero in size and individual nucleons locally interact with the WIMP, nuclear operators built from $\mathcal{O}_i$ are accompanied by an additional spatial operator $e^{-i\vec{q}\cdot\vec{x}(i)}$ where $x(i)$ is the location of the ith nucleon inside the nucleus.

Starting from Eq. (A2) and using the substitution rules for $\vec{v}^\perp$ and including a factor of $e^{-i\vec{q}\cdot\vec{x}_i}$, the interaction Lagrangian can be written as a sum of five distinct terms (nuclear electroweak operators) that only act on internal nucleon states. Their coefficients, on the other hand, act on WIMP in and out states. The WIMP-nucleus interaction can then be written as

$$\sum_{\tau=0,1}\{l_0^\tau S + l_0^{A\tau}T + \vec{l}_5^\tau\cdot\vec{P} + \vec{l}_M^\tau\cdot\vec{Q} + \vec{l}_E^\tau\cdot\vec{R}\}t^\tau(i) \tag{A7}$$

where

$$\begin{aligned}
S &= \sum_{i=1}^{A} e^{-i\vec{q}\cdot\vec{x}_i}\\
T &= \sum_{i=1}^{A}\frac{1}{2M}\left[-\frac{1}{i}\overleftarrow{\nabla}_i\cdot\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\cdot\frac{1}{i}\vec{\nabla}_i\right]\\
\vec{P} &= \sum_{i=1}^{A}\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i}\\
\vec{Q} &= \sum_{i=1}^{A}\frac{1}{2M}\left[-\frac{1}{i}\overleftarrow{\nabla}_i e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\frac{1}{i}\vec{\nabla}_i\right]\\
\vec{R} &= \sum_{i=1}^{A}\frac{1}{2M}[\overleftarrow{\nabla}_i\times\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\times\vec{\nabla}_i]
\end{aligned} \tag{A8}$$

and

$$\begin{aligned}
l_0^\tau &= c_1^\tau + ic_5^\tau\vec{S}_\chi\cdot\left(\frac{\vec{q}}{m_N}\times\vec{v}_T^\perp\right) + c_8^\tau(\vec{S}_\chi\cdot\vec{v}_T^\perp) + ic_{11}^\tau\frac{\vec{q}\cdot\vec{S}_\chi}{m_N} \qquad l_0^{A\tau} = -\frac{1}{2}\left[c_7^\tau + ic_{14}^\tau\left(\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right)\right]\\
\vec{l}_5 &= \frac{1}{2}\left[c_3^\tau i\frac{(\vec{q}\times\vec{v}_T^\perp)}{m_N} + c_4^\tau\vec{S}_\chi + c_6^\tau\frac{(\vec{q}\cdot\vec{S}_\chi)\vec{q}}{m_N^2} + c_7^\tau\vec{v}_T^\perp + ic_9^\tau\frac{(\vec{q}\times\vec{S}_\chi)}{m_N} + ic_{10}^\tau\frac{\vec{q}}{m_N}\right)\\
&\quad\times c_{12}^\tau(\vec{v}_T^\perp\times\vec{S}_\chi) + ic_{13}^\tau\frac{(S_\chi\cdot\vec{v}_T^\perp)\vec{q}}{m_N} + ic_{14}^\tau\left(\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right)\vec{v}_T^\perp + c_{15}^\tau\frac{(\vec{q}\cdot\vec{S}_\chi)(\vec{q}\times\vec{v}_T^\perp)}{m_N^2}\Bigg]\\
\vec{l}_M &= c_5^\tau\left(i\frac{\vec{q}}{m_N}\times\vec{S}_\chi\right) - \vec{S}_\chi c_8^\tau \qquad \vec{l}_E = \frac{1}{2}\left[c_3^\tau\frac{\vec{q}}{m_N} + ic_{12}^\tau\vec{S}_\chi - c_{13}^\tau\frac{(\vec{q}\times\vec{S}_\chi)}{m_N} - ic_{15}^\tau\frac{(\vec{q}\cdot\vec{S}_\chi)\vec{q}}{m_N^2}\right].
\end{aligned} \tag{A9}$$

The WIMP-nucleus amplitude, $\mathcal{M}$, can then be succinctly written as

$$\mathcal{M} = \sum_{\tau=0,1} \langle j_\chi, M_\chi; j_N, M_N | \{ l_0^\tau S + l_0^{A\tau} T + \vec{l}_5^\tau \cdot \vec{P} + \vec{l}_M^\tau \cdot \vec{Q} + \vec{l}_E^\tau \cdot \vec{R} \} t^\tau(i) | j_\chi, M_\chi; j_N, M_N \rangle. \tag{A10}$$

By using spherical decomposition, the internal nuclear operators S, T, P, Q, and R can be further rewritten in terms of standard nuclear electroweak responses as follows:

$$\begin{aligned}\mathcal{M} = \sum_{\tau=0,1} \langle j_\chi, M_{\chi f}; j_N, M_{Nf} | \Bigg(\sum_{J=0} \sqrt{4\pi(2J+1)}(-i)^J \left[l_0^\tau M_{J0;\tau} - i l_0^{A\tau} \frac{q}{m_N} \tilde{\Omega}_{J0;\tau}(q) \right] \\ + \sum_{J=1} \sqrt{2\pi(2J+1)}(-i)^J \sum_{\lambda\pm1} (-1)^\lambda \Big\{ l_{5\lambda}^\tau [\lambda \Sigma_{J-\lambda;\tau}(q) + i \Sigma'_{J-\lambda;\tau}(q)] \\ - i \frac{q}{m_N} l_{M\lambda}^\tau [\lambda \Delta_{J-\lambda;\tau}(q)] - i \frac{q}{m_N} l_{E\lambda}^\tau [\lambda \tilde{\Phi}_{J-\lambda;\tau}(q) + i \tilde{\Phi}'_{J-\lambda;\tau}(q)] \Big\} \\ + \sum_{J=0}^{\infty} \sqrt{4\pi(2J+1)}(-i)^J \left[i l_{50}^\tau \Sigma''_{J0;\tau}(q) + \frac{q}{m_N} l_{M0}^\tau \tilde{\Delta}''_{J0;\tau}(q) + \frac{q}{m_N} l_{E0}^\tau \tilde{\Phi}''_{J0;\tau}(q) \right] \Bigg) | j_\chi, M_{\chi i}; j_N, M_{Ni} \rangle \end{aligned} \tag{A11}$$

where there is an implicit sum over the nucleons,

$$\mathcal{O}_{JM;\tau}(q) \equiv \sum_{i=1}^{A} \mathcal{O}_{JM}(q\vec{x}_i) t^\tau(i), \tag{A12}$$

and the various electroweak responses are defined as

$$\begin{aligned} M_{JM}(q\vec{x}) &\equiv j_J(qx) Y_{JM}(\Omega_x) \\ \vec{M}_{JL}^M &\equiv j_L(qx) \vec{Y}_{JLM}(\Omega_x) \\ \Delta_{JM} &\equiv \vec{M}_{JJ}^M(qx_i) \cdot \frac{1}{q}\vec{\nabla}_i \\ \Sigma'_{JM} &\equiv -i \left\{ \frac{1}{q}\vec{\nabla}_i \times \vec{M}_{JJ}^M(q\vec{x}_i) \right\} \cdot \vec{\sigma}(i) \\ \Sigma''_{JM} &\equiv \left\{ \frac{1}{q}\vec{\nabla}_i M_{JM}(q\vec{x}_i) \right\} \cdot \vec{\sigma}(i) \\ \tilde{\Phi}'_{JM} &\equiv \left[\frac{1}{q}\vec{\nabla}_i \times \vec{M}_{JJ}^M(q\vec{x}_i) \right] \cdot \left[\vec{\sigma}(i) \times \frac{1}{q}\vec{\nabla}_i \right] \\ &\quad + \frac{1}{2}\vec{M}_{JJ}^M(q\vec{x}_i) \cdot \vec{\sigma}(i) \\ \Phi''_{JM} &\equiv i \left[\frac{1}{q}\vec{\nabla}_i M_{JM}(q\vec{x}_i) \right] \cdot \left[\vec{\sigma}(i) \times \frac{1}{q}\vec{\nabla}_i \right] \\ \Sigma_{JM} &\equiv \vec{M}_{JJ}^M(q\vec{x}_i) \cdot \vec{\sigma}(i) \\ \tilde{\Omega}_{JM} &\equiv \Omega_{JM}(q\vec{x}_i) + \frac{1}{2}\Sigma''_{JM}(q\vec{x}_i) \\ \tilde{\Phi}_{JM} &\equiv \Phi_{JM}(qx_i) - \frac{1}{2}\Sigma'_{JM}(qx_i) \\ \tilde{\Delta}''_{JM} &\equiv \Delta''_{JM}(qx_i) - \frac{1}{2}M_{JM}(qx_i) \end{aligned} \tag{A13}$$

where Y_{JM}, $\vec{Y}_{JLM}$ and j_j are spherical harmonics, vector spherical harmonics, and spherical bessel functions, respectively. We are only considering elastic transitions and assuming parity and CP as symmetries of the nuclear ground state. This eliminates some of the responses, and only $M, \Phi'', \Sigma', \Delta, \Sigma'', \tilde{\Phi}'$ survive. To calculate cross-sections, one needs to square the amplitude, average over initial spins, and sum over final spins. The matrix element squared for the nuclear portion of the amplitude has been made available by Fitzpatrick *et al.* [58], and codes have been supplied to calculate the full amplitude and rate [59].

APPENDIX B: VECTOR DARK MATTER

If the WIMP has spin 1, we find two extra operators that have not been considered previously. Specifically, the operators depend on the symmetric combination of polarization vectors, $S_{ij} = \frac{1}{2}(\epsilon_i^\dagger \epsilon_j + \epsilon_j^\dagger \epsilon_i)$. This necessitates a modification to the WIMP response functions by first modifying the ℓ coefficients given in Eq. (A9). Based on our nonrelativistic reduction for vector dark matter, the Lagrangian for vector dark matter and the nucleus, interacting via an uncharged scalar or vector mediator can be written in general as:

$$\begin{aligned}\mathcal{L}_{\text{vector}} = c_1\mathcal{O}_1 + c_4\mathcal{O}_4 + c_5\mathcal{O}_5 + c_8\mathcal{O}_8 + c_9\mathcal{O}_9 + c_{10}\mathcal{O}_{10} \\ + c_{11}\mathcal{O}_{11} + c_{14}\mathcal{O}_{14} + c_{17}\mathcal{O}_{17} + c_{18}\mathcal{O}_{18}\end{aligned} \tag{B1}$$

where we have defined $\mathcal{O}_{17} \equiv \frac{i\vec{q}}{m_N} \cdot \mathcal{S} \cdot \vec{v}_\perp$ and $\mathcal{O}_{18} \equiv \frac{i\vec{q}}{m_N} \cdot \mathcal{S} \cdot \vec{S}_N$ and the c_i's are given in Table IV. To decompose these new operators we replace $\vec{v}^\perp$ with the target velocity and the internucleon velocities and sum over nucleons. $\mathcal{O}_{17}$ can then be put into the form

$$\begin{aligned}\mathcal{O}_{17} \to \frac{i\vec{q}}{m_N} . \mathcal{S} . \Bigg[\vec{v}_T^\perp e^{-i\vec{q}.\vec{x}_i} - \sum_{i=1}^{A} \frac{1}{2M} \Bigg(-\frac{1}{i}\overleftarrow{\nabla}_i e^{-i\vec{q}\cdot\vec{x}_i} \\ + e^{-i\vec{q}\cdot\vec{x}_i} \frac{1}{i}\vec{\nabla}_i \Bigg)_{\text{int}} \Bigg].\end{aligned} \tag{B2}$$

$\mathcal{O}_{18}$ can be expanded as

$$\mathcal{O}_{18} \to \frac{1}{2}\frac{i\vec{q}}{m_N}\cdot \mathcal{S}\cdot \vec{\sigma}. \tag{B3}$$

Together, all the terms of $\mathcal{L}_{\mathrm{vector}}$ give rise to the following ℓ factors from Eq. (A9),

$$\begin{aligned}
\ell_0^\tau &= c_1^\tau + i\left(\frac{\vec{q}}{m_N}\times \vec{v}_T^\perp\right)\cdot \vec{S}_\chi c_5^\tau + (\vec{v}_T^\perp\cdot \vec{S}_\chi)c_8^\tau \\
&\quad + i\left(\frac{\vec{q}}{m_N}\cdot \vec{S}_\chi\right)c_{11}^\tau + i\left(\frac{\vec{q}}{m_N}\cdot \mathcal{S}\cdot \vec{v}_\perp^T\right)c_{17}^\tau \\
l_0^{A\tau} &= -i\left(\frac{\vec{q}}{2m_N}\cdot \vec{S}_\chi\right)c_{14}^\tau \\
\vec{l}_E^\tau &= 0 \\
\vec{l}_M^\tau &= i\left(\frac{\vec{q}}{m_N}\times \vec{S}_\chi\right)c_5^\tau - \vec{S}_\chi c_8^\tau - i\left(\frac{\vec{q}}{m_N}\cdot \mathcal{S}\right)c_{17}^\tau \\
\vec{l}_5^\tau &= \frac{1}{2}\vec{S}_\chi c_4^\tau + i\left(\frac{\vec{q}}{m_N}\times \vec{S}_\chi\right)c_9^\tau + \frac{1}{2}\left(i\frac{\vec{q}}{m_N}\right)c_{10}^\tau \\
&\quad + \frac{1}{2}\vec{v}_T^\perp\left(\frac{\vec{q}}{2m_N}\cdot \vec{S}_\chi\right)c_{14}^\tau + \frac{1}{2}\left(i\frac{\vec{q}}{m_N}\cdot \mathcal{S}\right)c_{18}^\tau.
\end{aligned} \tag{B4}$$

Based on the ℓ's above, the coefficients of the various nuclear responses are found by squaring the amplitude and then summing over spins. To simplify calculations, we choose a convenient basis for polarization vectors, $\epsilon_i^s = \delta_i^s$. Recall that the spin can then be written as the antisymmetric combination $iS_k = \epsilon_{ijk}\epsilon_i^\dagger \epsilon_j$. The WIMP responses unique to the vector case are then given by:

$$\begin{aligned}
R_M^{\tau\tau'} &= c_1^\tau c_1^{\tau'} + \frac{2}{3}\left(\frac{q^2}{m_N^2}v_T^{\perp 2}c_5^\tau c_5^{\tau'} + v_T^{\perp 2}c_8^\tau c_8^{\tau'}\right. \\
&\quad \left. + \frac{q^2}{m_N^2}c_{11}^\tau c_{11}^{\tau'} + \frac{q^2 v_T^{\perp 2}}{4m_N^2}c_{17}^\tau c_{17}^{\tau'}\right) \\
R_{\Phi''}^{\tau\tau'} &= 0 \\
R_{\Phi'' M}^{\tau\tau'} &= 0 \\
R_{\tilde{\Phi}'}^{\tau\tau'} &= 0 \\
R_{\Sigma''}^{\tau\tau'} &= \frac{1}{6}c_4^\tau c_4^{\tau'} + \frac{q^2}{4m_N^2}c_{10}^\tau c_{10}^{\tau'} + \frac{q^2}{12m_N^2}c_{18}^\tau c_{18}^{\tau'} \\
R_{\Sigma'}^{\tau\tau'} &= \frac{1}{6}c_4^\tau c_4^{\tau'} + \frac{q^2}{6m_N^2}c_9^\tau c_9^{\tau'} + \frac{q^2 v_T^{\perp 2}}{2m_N^2}c_{14}^\tau c_{14}^{\tau'} \\
&\quad + \frac{q^2}{24m_N^2}c_{18}^\tau c_{18}^{\tau'} \\
R_\Delta^{\tau\tau'} &= \frac{2}{3}\left(\frac{\vec{q}^2}{m_N^2}c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right) + \frac{q^2}{6m_N^2}c_{17}^\tau c_{17}^{\tau'} \\
R_{\Delta\Sigma'}^{\tau\tau'} &= \frac{2}{3}(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}).
\end{aligned} \tag{B5}$$

APPENDIX C: NONRELATIVISTIC REDUCTION

We find effective relativistic interaction Lagrangians by integrating out heavy mediators. We only keep the leading order interactions (suppressed by m or m^2). To the right of each operator is their nonrelativistic reduction expressed in terms of the operators in Table I with the coefficient derived from the Lagrangian parameters along with the relevant nucleon form factor. As multiple operators can have the same nonrelativistic limit, it is important to include the nucleon form factor at the relativistic level. If this is not performed, erroneous cancellations can occur.

For free spinors we use the Bjorken and Drell normalization and γ matrix conventions. In the nonrelativistic limit we make the following replacements:

$$\begin{aligned}
S &\to \frac{1_S}{\sqrt{m_S}} \qquad X_\mu \to \frac{\epsilon_\mu^s}{\sqrt{m_X}} \\
\chi &\to \sqrt{\frac{E+m_\chi}{2m_\chi}}\begin{pmatrix}\xi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m_\chi}\xi\end{pmatrix}
\end{aligned} \tag{C1}$$

where $s = 1, 2, 3$ are the different polarization states of the vector. $\xi = (10)^T$ is the left-handed Weyl spinor. The following Fierz transformation and gamma matrix identities were useful in the charged mediator cases, (a sign difference was found in the final identity when compared with [71]):

$$\begin{aligned}
(\bar{q}\chi)(\bar{\chi}q) &= -\frac{1}{4}\left[\bar{q}q\bar{\chi}\chi + \bar{q}\gamma^\mu q\bar{\chi}\gamma_\mu\chi + \frac{1}{2}\bar{q}\sigma^{\mu\nu}q\bar{\chi}\sigma_{\mu\nu}\chi\right. \\
&\quad \left. - \bar{q}\gamma^\mu\gamma^5 q\bar{\chi}\gamma_\mu\gamma^5\chi + \bar{q}\gamma^5 q\bar{\chi}\gamma^5\chi\right] \\
(\bar{q}\gamma^5\chi)(\bar{\chi}\gamma^5 q) &= -\frac{1}{4}\left[\bar{q}q\bar{\chi}\chi + \bar{q}\gamma^5 q\bar{\chi}\gamma^5\chi - \bar{q}\gamma^\mu q\bar{\chi}\gamma_\mu\chi\right. \\
&\quad \left. + \bar{q}\gamma^\mu\gamma^5 q\bar{\chi}\gamma_\mu\gamma^5\chi + \frac{1}{2}\bar{q}\sigma^{\mu\nu}q\bar{\chi}\sigma_{\mu\nu}\chi\right] \\
(\bar{q}\chi)(\bar{\chi}\gamma^5 q) &= -\frac{1}{4}[\bar{q}q\bar{\chi}\gamma^5\chi + \bar{q}\gamma^5 q\bar{\chi}\chi - \bar{q}\gamma^\mu q\bar{\chi}\gamma_\mu\gamma^5\chi \\
&\quad + \bar{q}\gamma^\mu\gamma^5 q\bar{\chi}\gamma_\mu\chi + i\epsilon_{\mu\nu\alpha\beta}\bar{q}\sigma^{\mu\nu}q\bar{\chi}\sigma^{\alpha\beta}\chi] \\
(\bar{q}\gamma_\mu\chi)(\bar{\chi}\gamma^\mu q) &= -\left[\bar{q}q\bar{\chi}\chi - \bar{q}\gamma^5 q\bar{\chi}\gamma^5\chi - \frac{1}{2}\bar{q}\gamma^\mu q\bar{\chi}\gamma_\mu\chi\right. \\
&\quad \left. - \frac{1}{2}\bar{q}\gamma^\mu\gamma^5 q\bar{\chi}\gamma_\mu\gamma^5\chi\right] \\
(\bar{q}\gamma_\mu\gamma^5\chi)(\bar{\chi}\gamma^\mu\gamma^5 q) &= -\left[-\bar{q}q\bar{\chi}\chi + \bar{q}\gamma^5 q\bar{\chi}\gamma^5\chi - \frac{1}{2}\bar{q}\gamma^\mu q\bar{\chi}\gamma_\mu\chi\right. \\
&\quad \left. - \frac{1}{2}\bar{q}\gamma^\mu\gamma^5 q\bar{\chi}\gamma_\mu\gamma^5\chi\right] \\
(\bar{q}\gamma_\mu\chi)(\bar{\chi}\gamma^\mu\gamma^5 q) &= -\left[\bar{q}q\bar{\chi}\gamma^5\chi - \bar{q}\gamma^5 q\bar{\chi}\chi + \frac{1}{2}\bar{q}\gamma^\mu q\bar{\chi}\gamma_\mu\gamma^5\chi\right. \\
&\quad \left. + \frac{1}{2}\bar{q}\gamma^\mu\gamma^5 q\bar{\chi}\gamma_\mu\chi\right]
\end{aligned} \tag{C2}$$

$$\sigma^{\mu\nu}\gamma^5 = \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}. \tag{C3}$$

All of the following operators are collected in terms of the coefficients of the NR operators, c_i, in Tables II–IV. The spinor case is in good agreement with the more complete set of relativistic operators given in [58,59]. In Tables VII–X, we list the non-relativistic reduction of the dark matter and quark bilinears for the cases of scalar dark matter, spin-$\frac{1}{2}$ dark matter with an uncharged mediator, spin-$\frac{1}{2}$ dark matter with a charged mediator, and vector dark matter, respectively.

APPENDIX D: QUARKS TO NUCLEONS

To go from the fundamental interactions of WIMPs with quarks to scattering from pointlike nucleons, one must evaluate the quark (parton) bilinears in the nucleons. For a full discussion see the appendix of [71] and [72]. We write the nucleon couplings in terms of the quark couplings times a form factor (in the limit of zero momentum transfer): The scalar

$$\begin{aligned}
\langle N_o|m_q\bar{q}q|N_i\rangle &\to f^N_{Tq}\bar{N}N\\
\langle N_o|\bar{q}\gamma^5 q|N_i\rangle &\to \Delta\tilde{q}^N\bar{N}\gamma^5 N\\
\langle N_o|\bar{q}\gamma^\mu q|N_i\rangle &\to \mathcal{N}^N_q\bar{N}\gamma^\mu N\\
\langle N_o|\bar{q}\gamma^\mu\gamma^5 q|N_i\rangle &\to \Delta^N_q\bar{N}\gamma^\mu\gamma^5 N\\
\langle N_o|\bar{q}\sigma^{\mu\nu} q|N_i\rangle &\to \delta^N_q\bar{N}\sigma^{\mu\nu} N
\end{aligned}$$

bilinear for light quarks can be evaluated from

$$\langle N|m_q\bar{q}q|N\rangle = m_N f^N_{Tq} \tag{D1}$$

while for the heavy quarks

TABLE VII. Nonrelativistic reduction of operators for a spin-0 WIMP.

Scalar Mediator	
$(S^\dagger S)(\bar{q}q)$	$\to (\frac{h_1^N g_1}{m_\phi^2})\mathcal{O}_1$
$(S^\dagger S)(\bar{q}\gamma^5 q)$	$\to (\frac{h_2^N g_1}{m_\phi^2})\mathcal{O}_{10}$
Vector Mediator	
$i(S^\dagger\partial_\mu S - \partial_\mu S^\dagger S)(\bar{q}\gamma^\mu q)$	$\to 0$
$i(S^\dagger\partial_\mu S - \partial_\mu S^\dagger S)(\bar{q}\gamma^\mu\gamma^5 q)$	$\to (\frac{2ig_4h_4^N}{m_G^2}\frac{m_N}{m_S})\mathcal{O}_{10}$
Charged Spinor Mediator	
$(S^\dagger S)(\bar{q}q)$	$\to \frac{y_1^\dagger y_1 - y_2^\dagger y_2}{m_Q m_S} f_T^N\mathcal{O}_1$
$(S^\dagger S)(\bar{q}\gamma^5 q)$	$\to i\frac{y_2^\dagger y_1 - y_1^\dagger y_2}{m_Q m_S}\tilde{\Delta}^N\mathcal{O}_{10}$

TABLE VIII. Operators for a spin-$\frac{1}{2}$ WIMP via a neutral mediator.

Scalar Mediator	
$\bar{\chi}\chi\bar{q}q$	$\to (\frac{h_1^N\lambda_1}{m_\phi^2})\mathcal{O}_1$
$\bar{\chi}\chi\bar{q}\gamma^5 q$	$\to (\frac{h_2^N\lambda_1}{m_\phi^2})\mathcal{O}_{10}$
$\bar{\chi}\gamma^5\chi\bar{q}q$	$\to (-\frac{h_1^N\lambda_2 m_N}{m_\phi^2 m_\chi})\mathcal{O}_{11}$
$\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	$\to (\frac{h_2^N\lambda_2 m_N}{m_\phi^2 m_\chi})\mathcal{O}_6$
Vector Mediator	
$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	$\to (-\frac{h_3^N\lambda_3}{m_G^2})\mathcal{O}_1$
$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	$\to (-\frac{2h_4^N\lambda_3}{m_G^2})(-\mathcal{O}_7 + \frac{m_N}{m_\chi}\mathcal{O}_9)$
$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$\to (-\frac{2h_3^N\lambda_4}{m_G^2})(\mathcal{O}_8 + \mathcal{O}_9)$
$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	$\to (\frac{4h_4^N\lambda_4}{m_G^2})\mathcal{O}_4$

TABLE IX. Nonrelativistic reduction of operators for a spin-$\frac{1}{2}$ WIMP via a charged mediator (after using Fierz identities).

Charged Scalar Mediator	
$\bar{\chi}\chi\bar{q}q$	$\to \frac{l_2^\dagger l_2 - l_1^\dagger l_1}{4m_\Phi^2} f^N_{Tq}\mathcal{O}_1$
$\bar{\chi}\chi\bar{q}\gamma^5 q$	$\to i\frac{l_1^\dagger l_2 - l_2^\dagger l_1}{4m_\Phi^2}\Delta\tilde{q}^N\mathcal{O}_{10}$
$\bar{\chi}\gamma^5\chi\bar{q}q$	$\to i\frac{l_2^\dagger l_1 - l_1^\dagger l_2}{4m_\Phi^2}\frac{m_N}{m_\chi} f^N_{Tq}\mathcal{O}_{11}$
$\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	$\to \frac{l_1^\dagger l_1 - l_2^\dagger l_2}{4m_\Phi^2}\frac{m_N}{m_\chi}\Delta\tilde{q}^N\mathcal{O}_6$
$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	$\to -\frac{l_1^\dagger l_1 + l_2^\dagger l_2}{4m_\Phi^2}\mathcal{N}^N_q\mathcal{O}_1$
$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$\to \frac{l_1^\dagger l_2 + l_2^\dagger l_1}{2m_\Phi^2}\mathcal{N}^N_q(\mathcal{O}_8 + \mathcal{O}_9)$
$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	$\to \frac{l_1^\dagger l_2 + l_2^\dagger l_1}{2m_\Phi^2}\Delta^N_q(\mathcal{O}_7 - \frac{m_N}{m_\chi}\mathcal{O}_9)$
$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	$\to -\frac{l_1^\dagger l_1 + l_2^\dagger l_2}{m_\Phi^2}\Delta^N_q\mathcal{O}_4$
$\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu} q$	$\to \frac{l_2^\dagger l_2 - l_1^\dagger l_1}{m_\Phi^2}\delta^N_q\mathcal{O}_4$
$\epsilon_{\mu\nu\alpha\beta}\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma^{\alpha\beta} q$	$\to \frac{l_2^\dagger l_1 - l_1^\dagger l_2}{m_\Phi^2}\delta^N_q(i\mathcal{O}_{10} - i\frac{m_N}{m_\chi}\mathcal{O}_{11} + 4\mathcal{O}_{12})$
Charged Vector Mediator	
$\bar{\chi}\chi\bar{q}q$	$\to \frac{d_2^\dagger d_2 - d_1^\dagger d_1}{4m_V^2} f^N_{Tq}\mathcal{O}_1$
$\bar{\chi}\chi\bar{q}\gamma^5 q$	$\to i\frac{d_2^\dagger d_1 - d_1^\dagger d_2}{4m_V^2}\Delta\tilde{q}^N\mathcal{O}_{10}$
$\bar{\chi}\gamma^5\chi\bar{q}q$	$\to i\frac{d_2^\dagger d_1 - d_1^\dagger d_2}{4m_V^2}\frac{m_N}{m_\chi} f^N_{Tq}\mathcal{O}_{11}$
$\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	$\to \frac{d_2^\dagger d_2 - d_1^\dagger d_1}{4m_V^2}\frac{m_N}{m_\chi}\Delta\tilde{q}^N\mathcal{O}_6$
$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	$\to \frac{d_2^\dagger d_2 + d_1^\dagger d_1}{8m_V^2}\mathcal{N}^N_q\mathcal{O}_1$
$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$\to -\frac{d_2^\dagger d_1 + d_1^\dagger d_2}{4m_V^2}\mathcal{N}^N_q(\mathcal{O}_8 + \mathcal{O}_9)$
$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	$\to \frac{d_2^\dagger d_1 + d_1^\dagger d_2}{4m_V^2}\Delta^N_q(\mathcal{O}_7 - \frac{m_N}{m_\chi}\mathcal{O}_9)$
$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	$\to -\frac{d_2^\dagger d_2 + d_1^\dagger d_1}{2m_V^2}\Delta^N_q\mathcal{O}_4$

TABLE X. Nonrelativistic reduction of operators for a spin-1 WIMP.

Scalar Mediator	
$X^\dagger_\mu X^\mu \bar{q}q$	$\rightarrow (\frac{b_1 h_1^N}{m_\phi^2})\mathcal{O}_1$
$X^\dagger_\mu X^\mu \bar{q}\gamma^5 q$	$\rightarrow (\frac{b_1 h_2^N}{m_\phi^2})\mathcal{O}_{10}$
Vector Mediator	
$(X^\dagger_\nu \partial_\mu X^\nu - \partial_\mu X^\dagger_\nu X^\nu)(\bar{q}\gamma^\mu q)$	$\rightarrow 0$
$(X^\dagger_\nu \partial_\mu X^\nu - \partial_\mu X^\dagger_\nu X^\nu)(\bar{q}\gamma^\mu \gamma^5 q)$	$\rightarrow (\frac{-3b_5 h_4^N}{m_G^2}\frac{m_N}{m_X})\mathcal{O}_{10}$
$\partial_\nu(X^{\nu\dagger}X_\mu + X^\dagger_\mu X^\nu)(\bar{q}\gamma^\mu q)$	$\rightarrow (\frac{\mathrm{Re}(b_6)h_3^N}{m_G^2}\frac{m_N}{m_X})(\mathcal{O}_5 + \mathcal{O}_6 - \frac{q^2}{m_N^2}\mathcal{O}_4)$
$\partial_\nu(X^{\nu\dagger}X_\mu + X^\dagger_\mu X^\nu)(\bar{q}\gamma^\mu \gamma^5 q)$	$\rightarrow (-\frac{2\mathrm{Re}(b_6)h_4^N}{m_G^2}\frac{m_N}{m_X})\mathcal{O}_9$
$\partial_\nu(X^{\nu\dagger}X_\mu - X^\dagger_\mu X^\nu)(\bar{q}\gamma^\mu q)$	$\rightarrow (-\frac{4\mathrm{Im}(b_6)h_3^N}{m_G^2}\frac{m_N}{m_X})\mathcal{O}_{17}$
$\partial_\nu(X^{\nu\dagger}X_\mu - X^\dagger_\mu X^\nu)(\bar{q}\gamma^\mu \gamma^5 q)$	$\rightarrow (\frac{4\mathrm{Im}(b_6)h_4^N}{m_G^2}\frac{m_N}{m_X})\mathcal{O}_{18}$
$\epsilon_{\mu\nu\rho\sigma}(X^{\nu\dagger}\partial^\rho X^\sigma + X^\nu \partial^\rho X^{\sigma\dagger})(\bar{q}\gamma^\mu q)$	$\rightarrow (\frac{\mathrm{Re}(b_7)h_3^N}{m_G^2}\frac{m_N}{m_X})\mathcal{O}_{11}$
$\epsilon_{\mu\nu\rho\sigma}(X^{\nu\dagger}\partial^\rho X^\sigma + X^\nu \partial^\rho X^{\sigma\dagger})(\bar{q}\gamma^\mu \gamma^5 q)$	$\rightarrow (\frac{\mathrm{Re}(b_7)h_4^N}{m_G^2}\frac{m_N}{m_X})(i\frac{q^2}{m_X m_N}\mathcal{O}_4 - i\frac{m_N}{m_X}\mathcal{O}_6 - 2\mathcal{O}_{14})$
$\epsilon_{\mu\nu\rho\sigma}(X^{\nu\dagger}\partial^\rho X^\sigma - X^\nu \partial^\rho X^{\sigma\dagger})(\bar{q}\gamma^\mu q)$	$\rightarrow (\frac{2\mathrm{Im}(b_7)h_3^N}{m_G^2})(\mathcal{O}_8 + \mathcal{O}_9)$
$\epsilon_{\mu\nu\rho\sigma}(X^{\nu\dagger}\partial^\rho X^\sigma - X^\nu \partial^\rho X^{\sigma\dagger})(\bar{q}\gamma^\mu \gamma^5 q)$	$\rightarrow (\frac{4\mathrm{Im}(b_7)h_4^N}{m_G^2})\mathcal{O}_4$
Charged Spinor Mediator	
$(X^\dagger_\mu X_\nu)(\bar{q}\gamma^\mu\gamma^\nu q)$	$\rightarrow (\frac{y_3^\dagger y_3 - y_4^\dagger y_4}{m_Q m_X})(f_{Tq}^N \mathcal{O}_1 + 2\delta_q^N \mathcal{O}_4)$
$(X^\dagger_\mu X_\nu)(\bar{q}\gamma^\mu\gamma^\nu\gamma^5 q)$	$\rightarrow (\frac{y_4^\dagger y_3 - y_3^\dagger y_4}{m_Q m_X})(i\Delta_{\tilde{q}}^N \mathcal{O}_{10} + i\delta_q^N \mathcal{O}_{11} - 2i\delta_q^N \mathcal{O}_{12} - 2i\delta_q^N \mathcal{O}_{18})$

$$\langle N|m_q\bar{q}q|N\rangle = \frac{2}{27}m_N F_{TG}^N = \frac{2}{27}m_N\left(1 - \sum_{q=u,d,s} f_{Tq}^N\right). \tag{D2}$$

Summing over all the quarks one finds

$$h_1^N = \sum_{q=u,d,s} h_1^q \frac{m_N}{m_q} f_{Tq}^N + \frac{2}{27} f_{TG}^N \sum_{q=c,b,t} h_1^q \frac{m_N}{m_q} \tag{D3}$$

The pseudoscalar bilinear was recently revisited in [72]:

$$h_2^N = \sum_{q=u,d,s} h_2^q \Delta\tilde{q}^N - \Delta\tilde{G}^N \sum_{q=c,b,t} \frac{h_2^q}{m_q}. \tag{D4}$$

The vector bilinear essentially gives the number operator:

$$h_3^N = \begin{cases} 2h_3^u + h_3^d & N = p \\ h_3^u + 2h_3^d & N = u. \end{cases} \tag{D5}$$

The pseudo-vector bilinear counts the contributions of spin to the nucleon (note that sometimes this coupling has a G_F factored out to make it dimensionless)

$$h_4^N = \sum_{q=u,d,s} h_4^q \Delta_q^N. \tag{D6}$$

Throughout this paper the following values are used (it should be noted that there are large uncertainties in these values) [71,72]:

$$\begin{aligned}
f_{Tu}^n &= 0.014 \qquad f_{Tu}^p = 0.02 \\
f_{Td}^n &= 0.036 \qquad f_{Td}^p = 0.026 \\
f_{Ts}^n &= 0.118 \qquad f_{Ts}^p = 0.118 \\
\Delta_u^n &= -0.427 \qquad \Delta_u^p = 0.842 \\
\Delta_d^n &= 0.842 \qquad \Delta_d^p = -0.427 \\
\Delta_s^n &= -0.085 \qquad \Delta_s^p = -0.085 \\
\Delta\tilde{u}^n &= -108.03 \qquad \Delta\tilde{u}^p = 110.55 \\
\Delta\tilde{d}^n &= 108.60 \qquad \Delta\tilde{d}^p = -107.17 \\
\Delta\tilde{s}^n &= -0.57 \qquad \Delta\tilde{s}^p = -3.37 \\
\Delta\tilde{G}^n &= 35.7\ \mathrm{MeV} \qquad \Delta\tilde{G}^p = 395.2\ \mathrm{MeV}.
\end{aligned} \tag{D7}$$

Assuming a universal coupling of the mediators to all quarks, the nucleon level couplings can then be written as,

$$\begin{aligned}
h_1^N &= f_T^N h_1 \qquad h_2^N = \tilde{\Delta}^N h_2 \\
h_3^N &= \mathcal{N}^N h_3 \qquad h_4^N = \Delta^N h_4
\end{aligned} \tag{D8}$$

where we have defined,

$$f_T^n = 11.93 \qquad f_T^p = 12.31 \qquad \tilde{\Delta}^n = -0.07 \qquad \tilde{\Delta}^p = -0.28 \qquad \mathcal{N}^n = 3 \qquad \mathcal{N}^p = 3$$
$$\Delta^n = 0.33 \qquad \Delta^p = 0.33 \qquad \delta^n = 0.564 \qquad \delta^p = 0.564. \tag{D9}$$

This introduces a small amount of isospin violation, and it is known that relaxing the assumption of universal couplings to quarks can lead to interesting isospin violating effects [72,73].

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